MATH4250 Game Theory
Exercise 4

Assignment 4: 1(a)(b), 2, 3(a)(c), 5 (Due: 27 March 2019 (Wednesday))

1. Find all Nash equilibria of the following bimatrix games. For each of the Nash equilibrium, find the payoff pair.

   (a) $\begin{pmatrix} (1,4) & (5,1) \\ (4,2) & (3,3) \end{pmatrix}$

   (b) $\begin{pmatrix} (5,2) & (2,0) \\ (1,1) & (3,4) \end{pmatrix}$

   (c) $\begin{pmatrix} (1,5) & (2,3) \\ (5,2) & (4,2) \end{pmatrix}$

2. The Brouwer's fixed-point theorem states that every continuous map $f : X \to X$ has a fixed-point if X is homeomorphic to a closed unit ball. Find a map $f : X \to X$ which does not have any fixed-point for each of the following topological spaces X. (It follows that the following spaces are not homeomorphic to a closed unit ball.)

   (a) X is the punched closed unit disc $D^2 \setminus \{0\} = \{(x, y) \in \mathbb{R}^2 : 0 < x^2 + y^2 \leq 1\}$

   (b) X is the unit sphere $S^2 = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1\}$

   (c) X is the open unit disc $B^2 = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}$

3. For each of the following bimatrices (A, B), find the values ν_A and ν_{B^T} of A and B^T respectively, and the Nash bargaining solution using $(\mu, \nu) = (\nu_A, \nu_{B^T})$ as the status quo point.

   (a) $\begin{pmatrix} (4,-4) & (-1,-1) \\ (0,1) & (1,0) \end{pmatrix}$

   (b) $\begin{pmatrix} (3,1) & (1,0) \\ (0,-1) & (2,3) \end{pmatrix}$

   (c) $\begin{pmatrix} (2,2) & (0,1) & (1,-1) \\ (4,1) & (-2,1) & (1,3) \end{pmatrix}$

   (d) $\begin{pmatrix} (6,4) & (0,10) & (4,1) \\ (8,-2) & (4,1) & (0,1) \end{pmatrix}$

4. Two broadcasting companies, NTV and CTV, bid for the exclusive broadcasting rights of an annual sports event. If both companies bid, NTV will win the bidding with a profit of \$20 (million) and CTV will have no profit. If only NTV bids, therell be a profit of \$50 (million). If only CTV bids, therell be a profit of \$40 (million). Find the Nash's solution to the bargaining problem.

5. Let $\mathcal{R} = \{(u, v) : v \geq 0 \text{ and } u^2 + v \leq 4\} \subset \mathbb{R}^2$. Find the arbitration pair $A(\mathcal{R}, (\mu, \nu))$ using the following points as the status quo point (μ, ν).

   (a) $(0, 0)$

   (b) $(0, 1)$

6. Let $\mathcal{R} \subset \mathbb{R}^2$ be a closed and bounded convex set, $(\mu, \nu) \in \mathcal{R}$ and $(\alpha, \beta) = A(\mathcal{R}, (\mu, \nu))$ be the arbitration pair with $\alpha \neq \mu$. Suppose the boundary of $\mathcal{R}$ is given, locally at (α, β), by the graph of a differentiable function $f(x)$ with $f(\alpha) = \beta$. Prove that $f'(\alpha)$ is equal to the negative of the slope of the line joining (μ, ν) and (α, β).